# DY 6: Statistical physics of complex networks II

Time: Monday 14:30-16:15

## DY 6.1 Mon 14:30 H3

Fluctuation-dissipation relations in complex networks — AGATA FRONCZAK, PIOTR FRONCZAK, and •JANUSZ HOLYST — Faculty of Physics and Center of Excellence for Complex Systems Research, Warsaw University of Technology, Koszykowa 75, PL-00-662 Warsaw, Poland

In this paper, we study fluctuations over several ensembles of maximum-entropy random networks. We derive several fluctuationdissipation relations characterizing the susceptibilities of different networks to changes in external fields. In the case of networks with a given degree sequence, we argue that the scale-free topologies of real-world networks may arise as a result of the self-organization of real systems into sparse structures with low susceptibility to random external disruptions. We also show that the ensembles of networks with a given degree sequence and networks characterized by two-point correlations are equivalent to random networks with hidden variables.

#### DY 6.2 Mon 14:45 H3

Ising model on two connected Barabasi-Albert networks — •KRZYSZTOF SUCHECKI and JANUSZ HOLYST — Faculty of Physics and Center of Excellence for Complex Systems Research, Warsaw University of Technology, Koszykowa 75, PL-00-662 Warsaw, Poland

We have investigated analytically the behavior of Ising model on two connected Barabasi-Albert networks. Depending on the temperature and the number of inter-network connections, the system can order in one or two possible phases. In the first phase both networks are ordered paralelly. In the second phase there is a ferromagnetic order inside each networks and antiparallel order between them.

At low temperatures both phases can exist depending on initial conditions. At a certain critical temperature Tc-, the antiparallel state becomes unstable and one of the networks reverses its magnetization. This is a first order phase transition. At a higher temperature Tc+ the networks do not maintain common ordering, and the system becomes paramagnetic. This is a standard second order phase transition.

Both critical temperatures Tc- and Tc+ depend on network size and internetwork connections ratio p, defined as amount of internetwork links to intranetwork links. While Tc+ increases with p in linear fashion, the dependence Tc- on p is much more complex, but strictly decreasing. At p=1 the temperature Tc=0.

Analytic calculations of critical temperatures, based on a mean field approach, are in qualitative agreement with Monte Carlo simulations of above systems.

#### DY 6.3 Mon 15:00 H3

**Evolution of a Population of Boolean Networks** — •TAMARA MIHALLEV and BARBARA DROSSEL — Institut für Festkörperphysik, Technische Universität Darmstadt, Deutschland

Boolean network models share several dynamical features with real genetic regulatory networks, although they are much simpler. Studying the evolution of such models may therefore help to gain insights into how evolutionary forces shape real genetic networks. Starting with a population of random Boolean networks with canalyzing update functions, we obtain subsequent generations of this population by producing offspring according to the fitness criterion "robustness" (i.e. the probability that an attractor remains stable under small perturbations), and by performing in some offspring mutations that change the logical structure and the topology of the networks. The fitness landscape has a huge neutral space with maximum fitness. We study the features of the evolutionary process and the properties of the evolved populations as function of the mutation rate and the strength of selection. Quantities investigated are the speed of evolution, the homogeneity of the population, and the mean and the maximum fitness of the population.

DY 6.4 Mon 15:15 H3 Scaling and criticality in finite dynamical networks at the SP limit — •THIMO ROHLF<sup>1</sup>, NATALI GULBAHCE<sup>2</sup>, and CHRISTOF TEUSCHER<sup>3</sup> — <sup>1</sup>Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501, USA — <sup>2</sup>LANL, Center for Nonlinear Studies, MS B258, Los Alamos, NM 87545, USA — <sup>3</sup>LANL, Advanced Computing Laboratory, MS B287, Los Alamos, NM 87545, USA

It has been shown that both Random Boolean Networks (RBN) and

Location: H3

Random Threshold Networks (RTN) exhibit a order-disorder transition at a critical average connectivity  $K_c$  in the thermodynamical limit [1,2]. Looking at the statistical distributions of damage spreading for both RBN and RTN, we go beyond this mean-field approximation.

We study the scaling properties of damage size distributions as a function of system size N and initial perturbation size d(t = 0) in the sparse percolation (SP) limit (i.e.  $d(t = 0)/N \rightarrow 0$  for large N). We present evidence that another characteristic point,  $K_s$  exists for finite systems, where the expectation value of damage spreading in the network is independent of N. We find that damage distributions strongly depend on the order of averages taken over dynamics and network ensembles, possibly limiting the validity of mean-field predictions.

Finally, we discuss the implications of our findings for evolutionary processes and learning applied to networks which solve specific computational tasks.

[1] Derrida, B. and Pomeau, Y. (1986), Europhys. Lett., 1, 45-49

[2] Rohlf, T. and Bornholdt, S. (2002), Physica A 310, 245-259

### DY 6.5 Mon 15:30 H3

Rich dynamical behavior of simple canalyzing Kauffman networks — •FLORIAN GREIL and BARBARA DROSSEL — Institut für Festkörperphysik, Technische Universität Darmstadt, Deutschland

We investigate Threshold Random Boolean Networks with K = 2 inputs per node, which are equivalent to Kauffman networks, with only part of the canalyzing functions as update functions. While these models should be critical according to the simplest consideration, it turns out that they show in reality a rich variety of behaviors as the relative weights of the different canalyzing functions are changed. We see frozen, critical and chaotic behavior, as well as oscillations with period 2, and several critical points between these different regimes. The results are supported by analytical calculations and computer simulations.

#### DY 6.6 Mon 15:45 H3

**Emergence of networks from optimizing local interaction** — •MICHAEL KÖNIG, STEFANO BATTISTON, and FRANK SCHWEITZER — Chair of Systems Design, ETH Zurich, Kreuzplatz 5, 8032 Zurich, Switzerland

We model evolving complex networks in which agents select their interactions with other agents on the basis of a local nonlinear utility function and study the resulting global network structure. Agents can increase each others utility as catalytic processes on a directed dynamic network. Two cases are discussed: (i) Agents can either bilaterally increase each others utility (direct reciprocity) or (ii) unilaterally increase the utility of other agents and hope that they will benefit from the support of another agent (indirect reciprocity). Direct reciprocity corresponds to a cycle of order k=2 in the network, while indirect reciprocity corresponds to a cycle of order k > 2. The emergence of an autocatalytic set (ACS) is the driving process of growth and sustainability in our model. An ACS is a subgraph of a network, each of whose nodes has at least one incoming link belonging to the same subgraph. The core of an ACS consists of a closed cycle. We argue that in case (i) the network evolves towards a random graph consisting of bilateral links, while in case (ii) an ACS will only form if agents percive an extra profit from being part of a directed cycle.

DY 6.7 Mon 16:00 H3 Networks interacting with matter — •BARTLOMIEJ WACLAW<sup>1</sup>, ZDZISLAW BURDA<sup>2</sup>, and WOLFHARD JANKE<sup>1</sup> — <sup>1</sup>Institut für Theoretische Physik, Universität Leipzig, Augustusplatz 10/11, 04109 Leipzig, Germany — <sup>2</sup>M. Smoluchowski Institute of Physics, Jagellonian University, Reymonta 4, 30-059 Krakow, Poland

Many processes taking place in nature can be described in terms of complex networks. They can be divided into three classes: the evolution of networks, the dynamics on networks, and the dynamical interaction between the network topology and matter degrees of freedom living on the network. The latter process has two characteristic time scales: the first one related to the dynamics of the matter, and the second one related to the dynamics of network connections. Many results have been found for the case where these two scales differ significantly, but less is known what happens when they are comparable. Here we would like to propose a simple model, being a composition of the Zero Range Process and a dynamically changing network, which allows for studying the regime where the dynamics of network topology significantly affects the dynamics of the processes on the network. We show that the model possesses a phase transition between a condensed and uncondensed state resulting from the interplay of these two dynamical degrees-of-freedom.